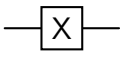
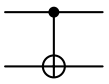
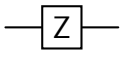
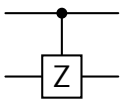
13 June 2025

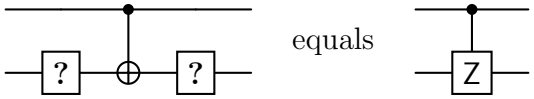
Problem Solving Session - I

Question 1: Recall the following

X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	
C-X	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	
Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	
C-Z	$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$	

(a) Write down the matrix for the controlled Z gate.

(b) In the following circuit, what should be "**?**" so that we can get a controlled Z gate from a controlled X gate ?

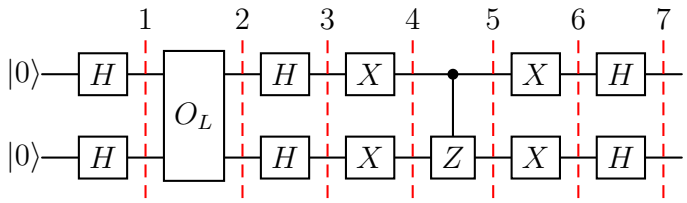


Question 2: Recall that $H^{\otimes n} = H \underbrace{\otimes \ldots\ldots \otimes}_{\text{n times}} H$. This is the same as applying H individually on each of the n registers.

1. What is $H^{\otimes 3} |010\rangle$?
2. Let $x_1 \in \{0,1\}$ and $H|x_1\rangle = \alpha|0\rangle + \beta|1\rangle$. Express α and β as a function of x_1.
3. Let $x_1, x_2 \in \{0,1\}$ and $H^{\otimes 2}|x_1x_2\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$.Express $\alpha_{00}, \alpha_{01}, \alpha_{10}$ and α_{11} as a function of x_1 and x_2.
4. Let $x_1, \ldots, x_n \in \{0,1\}$ and $H^{\otimes n}|x_1x_2\ldots x_n\rangle = \sum_{y\in\{0,1\}^n} \alpha_y |y\rangle$. For each $y \in \{0,1\}^n$, express α_y as a function of $x_1, \ldots, x_n$ [*Note: Your expression of α_y must depend on $x_1, \ldots, x_n$ as well as $y_1, \ldots, y_n$*].

Question 3: Let $L = \{00, 01, 10, 11\}$ with the state 01 be marked. Let O_L be a phase oracle that implements it. This means, $O_L |xy\rangle = \begin{cases} -|01\rangle & \text{if } xy = 01 \\ |xy\rangle & \text{otherwise} \end{cases}$

The following circuit implements a Grover search for the element 01 in L.



Write down the value of the state at all the slices from 1 through 7 with $|00\rangle$ being the initial state.