17 June 2025

Problem Solving Session - III

Question 1: Consider an operator that maps $R_k|0\rangle = |0\rangle$ and $R_k|1\rangle = e^{2\pi/2^k}|1\rangle$.

(a) Prove that R_k is unitary. Recall that U is unitary if $U^*U = I$.

(b) Write down the eigenstates, eigenvalues and their corresponding phases of R_k.

Question 2: Let c denote an n bit integer whose binary representation is given by $c_1c_2\ldots c_n$ with c_n being the least significant bit.

(a) Argue that

$$e^{2\pi c/2^n}|c\rangle = \otimes_{j=1}^{n} e^{2\pi c_j/2^j}|c_j\rangle$$

(b) Use R_k gates to implement

$$|c\rangle \to e^{2\pi c/2^n}|c\rangle$$

Question 3: Let a be an n bit integer, $N = 2^n$ and b denote $(a+1) \mod N$. Let $|\widehat{a}\rangle$ denote $QFT|a\rangle$ and let $|\widehat{b}\rangle$ denote $QFT|b\rangle$.

(a) Design a quantum circuit that does

$$|\widehat{a}\rangle \to |\widehat{b}\rangle$$

(b) Using the above design a quantum circuit that does

$$|a\rangle \to |a+1 \mod N\rangle$$