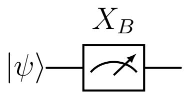
18 June 2025

Problem Solving Session - IV

Consider the following Bomb measurement operator acting on a single qubit state $|\psi\rangle$ as follows:



$$\begin{cases} |0\rangle & \text{with no explosion} \\ |1\rangle & \text{with explosion} \end{cases}$$

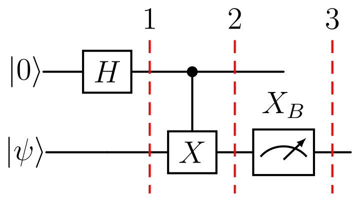
If the outcome is $|0\rangle$ upon measurement, then there is no explosion and the state will be in $|0\rangle$. If the outcome is $|1\rangle$, then there is explosion. In general, for $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, the outcomes are

$$\begin{cases} |0\rangle & \text{with probability } |\alpha|^2 \\ \text{explodes} & \text{with probability } |\beta|^2 \end{cases}$$

It has been guaranteed that $|\psi\rangle = |0\rangle$ or $|1\rangle$. The goal is to determine the state of $|\psi\rangle$ without exploding the Bomb (during the measurement).

Question 1: What happens when one measures $|\psi\rangle$ using X_B directly ?

Question 2: Consider the following circuit:



Upon measurement, it is possible that the bomb can explode. If that did not happen and the measurement gave a state $|b\rangle$. Then, output "No bomb" if $b = 0$ and "Bomb" if $b = 1$.

(a) Write down the states at all the slices marked.

(b) Compute the probability that it outputs "Bomb" without exploding the Bomb.