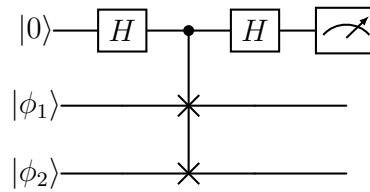


Problem Solving Session - V

Question 0: Describe the behaviour of the controlled SWAP (Fredkin) gate.

Question 1: You are given two single qubits $|\phi_1\rangle$ and $|\phi_2\rangle$ via unitaries U_1 and U_2 . The goal is to check if $|\phi_1\rangle$ is same as that of $|\phi_2\rangle$.

Consider the following circuit:



- (a) Write down the state at all the slices.
- (b) Write down the probability of getting $|1\rangle$ in the first gate on measurement. (Your expression should appear as a function of $\langle\phi_1|\phi_2\rangle$)

This test is also called as a SWAP test.

Question 2: Suppose that D_1 and D_2 be operators that corresponds to two probability distributions given by $D_1|0\rangle = \sqrt{p_0}|0\rangle + \sqrt{1-p_0}|1\rangle$ and $D_1|1\rangle = \sqrt{p_1}|0\rangle + \sqrt{1-p_1}|1\rangle$. Similarly, $D_2|0\rangle = \sqrt{q_0}|0\rangle + \sqrt{1-q_0}|1\rangle$ and $D_2|1\rangle = \sqrt{q_1}|0\rangle + \sqrt{1-q_1}|1\rangle$.

Let $|\psi_1\rangle = \frac{D_1|0\rangle + D_1|1\rangle}{\sqrt{2}}$ and $|\psi_2\rangle = \frac{D_2|0\rangle + D_2|1\rangle}{\sqrt{2}}$. Use the previous questions to tell D_1 and D_2 apart.