

QUIZ 1

Boolean strings. We write $\{0, 1\}^3$ for the set of all 0-1 strings with three symbols each, that is,

$$\{0, 1\}^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}.$$

Similarly, we write $\{0, 1\}^n$ for the set of all 0-1 strings with n symbols each.

Question 1 (two minutes): How many strings are there in the set $\{0, 1\}^n$?

Boolean functions. A Boolean function with 3 input bits and 2 output bits is a function from the set $\{0, 1\}^3$ to the set $\{0, 1\}^2$.

Question 2 (3 minutes): How many such functions are there?

Boolean circuits. Consider the Boolean function EQ from $\{0, 1\}^2$ to $\{0, 1\}$ given by

$$00 \mapsto 1$$

$$01 \mapsto 0$$

$$10 \mapsto 0$$

$$11 \mapsto 1$$

That is $\text{EQ}(x_1, x_2) = 1$ precisely when $x_1 = x_2$. Build a circuit with two inputs x_1 and x_2 and one output that computes EQ correctly.

Question 3 (5 minutes). Draw a Boolean circuit consisting of AND, OR and NOT gates, with two inputs bits labelled x_1 and x_2 and one output bit such that for all choices $(x_1, x_2) \in \{0, 1\}^2$, the output of the circuit is exactly $\text{EQ}(x_1, x_2)$.

