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QUIZ 7

Suppose registers A (with Alice) and B (with Bob) are in the state

$$|\psi_0\rangle = \alpha\,|00\rangle_{AB} + \beta\,|11\rangle_{AB}\,.$$

Here α and β are unknown to Alice and Bob. Note that (we will explain this in class) unlike in the classical randomized case, the reduced (marginal) state of register B is not exactly $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. Alice would like to help Bob obtain $|\psi\rangle$ in his register B.

Question 1: Suppose Alice applies the Hadamard gate to the register A. By grouping terms appropriately, write the resulting state in the form

$$|\psi_1\rangle_{AB} = \frac{1}{\sqrt{2}}\,|0\rangle_A\,(a\,|0\rangle_B + b\,|1\rangle_B) + \frac{1}{\sqrt{2}}\,|1\rangle_A\,(c\,|0\rangle_B + d\,|1\rangle_B).$$

Write a, b, c, d in terms of α and β?

$$a = \_\_\_\_\_, b = \_\_\_\_\_, c = \_\_\_\_\_, d = \_\_\_\_\_.$$

Now assume Alice measures register A.

Question 2: Suppose Alice's measurement yields a 0. Alice needs to tell Bob to apply a certain single-qubit gate G_0 on register B so that register B's state becomes $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. What should gate G_0 be?

$$G_0 = \_\_\_\_\_$$

Question 3: Next, suppose Alice's measurement yields a 1. What gate G_1 must Alice ask Bob to apply on B so that register B's state becomes $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. What should gate G_1 be?

$$G_1 = \_\_\_\_\_$$