20 June 2025

Quiz 9

Let $G = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. Consider the following circuit

Let $|\psi\rangle$ be the output state of the circuit expressed as $\frac{1}{\sqrt{2}}|0\rangle|\psi_0\rangle + \frac{1}{\sqrt{2}}|1\rangle|\psi_1\rangle$.

Question 1 Let G' be an operator such that

$$G'|0\rangle \to |\psi_0\rangle$$
$$G'|1\rangle \to |\psi_1\rangle$$

Write down a matrix corresponding to the operator G'.

Question 2 For an angle $\theta \in [-\pi, \pi]$, let $|u\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$ and $|u^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$.

Consider the state $(\frac{1}{\sqrt{2}}|u\rangle|u\rangle + \frac{1}{\sqrt{2}}|u^\perp\rangle|u^\perp\rangle)$. Express this state in the form $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ and write down the values of a, b, c and d.